SHOCK WAVES IN SOILS AND IN WATER NEAR THE POINT OF EXPLOSION

G. M. Lyakhov, V. N. Okhitin, and A. G. Chistov

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This paper gives a solution to the problem of the propagation of a plane shock wave in soils and in water; the solution was obtained by the method of characteristics using an electronic computer. Here, the soils were regarded as multicomponent media, in accordance with a previously proposed model [1, 2]. A comparison is made between the parameters of the waves and the dimensions of the gas cavity in soils with a different content of their components and in water.

1. Model of a Soil as a Multicomponent Medium and Its Experimental Confirmation

A model of soils and rocks, including solid particles, water, and gas as multicomponent media, has been proposed previously [1, 2]. It is assumed that the gaseous, liquid, and solid components of the medium are compressed in accordance with the same law, but in a free state, i.e., respectively, in accordance with the equations

$$p = p_0 \left(\frac{\rho}{\rho_1}\right)^{\gamma_1} \tag{1.1}$$

$$p = p_0 + \frac{\rho_2 c_2^2}{\gamma_2}\left[\left(\frac{\rho}{\rho_2}\right)^{\gamma_2} - 1\right] \tag{1.2}$$

$$p = p_0 + \frac{\rho^2 c_3^2}{\gamma_3}\left[\left(\frac{\rho}{\rho_3}\right)^{\gamma_3} - 1\right] \tag{1.3}$$

where ρ_1, ρ_2, and ρ_3 are the densities, and c_1, c_2, and c_3 are the velocities of sound in the components, at $p = p_0$.

The equation of dynamic compressibility of a three-component medium is written in the form

$$\frac{\rho_0}{\rho} = \alpha_1 \left(\frac{p}{p_0}\right)^{-\gamma_1^{-1}} + \alpha_2\left[\frac{\gamma_2 (p - p_0)}{\rho_1 c_2^2} + 1\right]^{-\gamma_2^{-1}} + \alpha_3\left[\frac{\gamma_3 (p - p_0)}{\rho_3 c_3^2} + 1\right]^{-\gamma_3^{-1}} \tag{1.4}$$

where α_1, α_2, α_3 are the contents of the gaseous, liquid, and solid components in the medium, by volume; ρ_0 is the density of the medium at $p = p_0$

$$\alpha_1 + \alpha_2 + \alpha_3 = 1, \quad \rho_0 = \alpha_1\rho_1 + \alpha_2\rho_2 + \alpha_3\rho_3 \tag{1.5}$$

The strength and the compressibility of the skeleton are not taken into account in this model; therefore, it is applicable only at pressures exceeding some value p*, above which, the compressibility of the skeleton may be neglected. In accordance with experimental data [1, 3], the value of p* corresponds

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TABLE 1

	Experiment		First calculation		Second calculation		Third calculation	
$p \cdot 10^{-3}$, kg/cm^2	68	107	68	107	68	107	68	107
ρ, g/cm^3	2.76	2.92	2.63	2.77	2.60	2.73	2.70	2.82
ρ/ρ_0	1.36	1.44	1.31	1.36	1.28	1.35	1.33	1.39

TABLE 2

	Experiment		Third calculation	
$p \cdot 10^{-3}$, kg/cm^2	32.8	66	32.8	66
ρ, g/cm^3	2.83	3.07	2.72	2.98
ρ/ρ_0	1.32	1.43	1.27	1.39

approximately to atmospheric pressure with $\alpha_1 = 0$, to several atmospheres with $\alpha_1 = 0.02$–0.04, to $5 \cdot 10^3$ kg/cm^2 at $\alpha_1 = 0.012$–0.018, and to $20 \cdot 10^3$ kg/cm^2 at $\alpha_1 = 0.2$–0.3. The value of p^* exceeds by several times the value of the pressure at which the volumetric deformation of the soil is equal to the volumetric content of the gaseous component. At $p < p^*$, when the compressibility of the soils is determined by the compressibility of the skeleton, models of an elastic–plastic medium are applied to these soils [4, 5]. Experiments show that, in the solution of certain wave problems at $p < p^*$, not only the elastic and plastic, but also the viscous properties of the soil, must be taken into account. Such a model was proposed in [3].

At pressures of tens and hundreds of thousands of atmospheres, near the point of explosion of an explosive, we may expect deviations in the properties of the soils from the model of a multicomponent medium as the result of possible phase transformations of the solid component and of a change in Eq. (1.3). The experimental data of [6] permit verifying the correspondence of the properties of the soil to the model of a multicomponent medium at these pressures. Table 1 gives experimental values of the density of clay B_{20}, corresponding to dynamic loading at two pressures [6] and to calculations using Eq. (1.4). The properties of clay B_{20} are: $\rho_0 = 2.03$ g/cm^3; moisture content by weight w = 20%; $\rho_3 = 2.7$ g/cm^3, which corresponds to $\alpha_1 = 0.035$, $\alpha_2 = 0.338$, $\alpha_3 = 0.627$.

Table 2 gives calculated and experimental values of the density of clay B_4, with the properties: $\rho_0 = 2.15$ g/cm^3, w = 4%, $\rho_3 = 2.7$ g/cm^3, which corresponds to $\alpha_1 = 0.146$, $\alpha_2 = 0.088$, $\alpha_3 = 0.766$.

To verify the degree of effect of the values of $\rho_i c_i$ and γ_i used on the values of the density, three calculations were made:

1) $\gamma_2 = 6.29$, $c_2 = 1620$ m/sec, $\gamma_3 = 4$, $c_3 = 4500$ m/sec. The values of γ_2 and of the nominal velocity of sound in water, c_2, were taken from [7], and the values of c_3 and γ_3 from [2];

2) $\gamma_2 = 7$, $c_2 = 1500$ m/sec, $\gamma_3 = 7$, $c_3 = 4500$ m/sec;

3) $\gamma_2 = 6.29$, $c_2 = 1620$ m/sec, $\gamma_3 = 4$, $c_3 = 3780$ m/sec. The values of γ_3 and c_3 were taken from experiments on the dynamic compression of quartz [8].

In all cases, the remaining values were identical: $\rho_1 = 12 \cdot 10^{-4}$ g/cm^3, $\rho_2 = 1$ g/cm^3, $\rho_3 = 2.7$ g/cm^3.

It follows from the data of Tables 1 and 2 that the calculated values of the density of soils, corresponding to pressures of tens of thousands of atmospheres, at the chosen values of γ_i and c_i, differ between themselves and from the experimental values, by several percent.

In addition to the equation for the compression of a medium, for the solution of plane one-dimensional wave problems, we need to know the unloading equation. Experiments [6] show that in clays B_{20} and B_4, unloading, at pressures of tens of thousands of atmospheres, takes place along a line close to the loading line. The velocity of sound, calculated under the assumption that the loading and unloading lines coincide (1.4), is equal to 5420 m/sec in clay B_{20}; experiment yields a value of 5610 m/sec. For B_4, the difference is somewhat greater. Thus, close to the point of explosion, clays with the above content of their components can be regarded as media whose compression and unloading take place in accordance with Eq. (1.4).

2. Initial Parameters

In the solution of wave problems, schemes for the real and instantaneous detonation of explosives are applied. For flat charges with a small thickness (on the order of cm) and a large area (on the order of m^2), with initiation of the explosion at one point, at distances sufficiently far removed from this point, the detonation takes place simultaneously over the whole thickness of the charge. In this case, the scheme for instantaneous detonation corresponds more closely to the actual process than the scheme of a real detonation, which assumes the approach of the detonation wave along a normal to the surface of the charge.

TABLE 3

Media	Characteristics of media				Dimensionless parameters			Dimensional parameters		
	α_1	α_2	α_3	ρ_0, g/cm^3	p_T	u_T	V_T	p_T, kg/cm^2	u_T, m/sec	ρ, g/cm^3
First	0	0.4	0.6	1.99	0.578	0.166	0.66	$54\cdot10^3$	696	2.42
Second	0.02	0.33	0.65	2.05	0.582	0.165	0.68	$54.6\cdot10^3$	692	2.50
Third	0	1	0	1	0.408	0.258	1.10	$38.3\cdot10^3$	1080	1.45

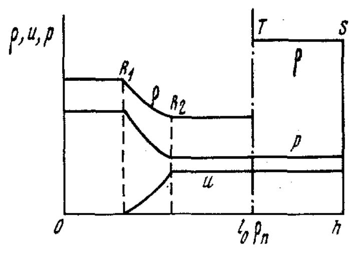

Fig. 1

Let us find the initial parameters of the wave. We consider a flat charge. Edge effects are not taken into account. The thickness of the charge is $2l_0$. On both sides of the charge the medium is soil or water. At t = 0, the charge detonates instantaneously. By virtue of symmetry, the process is considered from one side, from the middle of the charge. We use Lagrange variables (h is the mass, t the time). The origin of coordinates is the point of symmetry. To the right of the boundary of the charge ($h = l_0\rho_n$) a stationary shock wave S is propagated, and, to the left along the products of the detonation, a rarefaction wave R_1 (Fig. 1). These waves are separated by a region of detonation products with constant parameters. We denote the boundary between the rarefaction wave and this region by R_2, and the boundary between the detonation products and the medium by T (a contact discontinuity). At T, the velocity of the particles and the pressure are identical on both sides, while the density undergoes a discontinuity. We denote by u_T and p_T the velocity of the particles and the pressure at the contact discontinuity. These are the initial values for a wave in a medium

$$p = p_n (\rho / \rho_n)^k \tag{2.1}$$

With instantaneous detonation, as is well known

$$p_n = \frac{\rho_n D_n^2}{2(k+1)}, \quad c_n = \sqrt{kp_n\rho_n^{-1}} \tag{2.2}$$

where D_n, ρ_n, and k are the given properties of the explosive; D_n is the detonation rate; ρ_n is the density; k is the isentropic index; c_n is the velocity of sound.

The equations of motion in the variables h, t have the form

$$\frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad \frac{\partial u}{\partial h} - \frac{\partial V}{\partial t} = 0 \tag{2.3}$$

In a rarefaction wave, the flow is determined by the equations

$$u = \pm\int \sqrt{-\frac{dV}{dp}}\, dp + \text{const}, \quad h = \pm\sqrt{\left(\frac{dV}{dp}\right)^{-1}}\, t + \varphi(u), \quad V = \rho^{-1} \tag{2.4}$$

which are special solutions of the system (2.3).

The arbitrary function $\varphi(u)$ and the constant quantity are determined from the boundary conditions: at R_1, we have $p = p_n$, $u = 0$; at R_2, we have $u = u_T$, $p = p_T$, i.e., the same values as at the contact discontinuity and the shock wave.

Taking account of the boundary conditions and of the equation of state (1.4) at k = 3, we obtain in the rarefaction wave

$$h = -At + l_0\rho_n, \quad u + c = c_n$$

$$\frac{p}{p_n} = \left(\frac{-h + l_0\rho_n}{A_n t}\right)^{3/2}, \quad \frac{c}{c_n} = \left(\frac{-h + l_0\rho_n}{A_n t}\right)^{1/2}, \quad A = c\rho, \quad A_n = c_n\rho_n \tag{2.5}$$

$$\frac{u}{c} = 1 - \left(\frac{p}{p_n}\right)^{1/3} \tag{2.6}$$

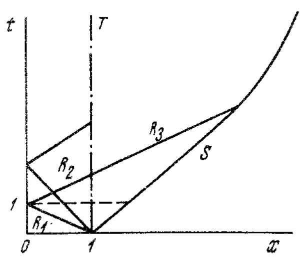

Fig. 2

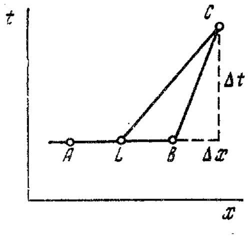

Fig. 3

At the front of the shock wave, the following relationships are satisfied

$$u = h_s^{\cdot}(V_0 - V), \quad p - p_0 = h_s^{\cdot 2}(V_0 - V) \qquad (2.7)$$
$$u^2 = (p - p_0)(V_0 - V)$$

where $h_s^{\cdot}$ is the velocity of the front of the shock wave; $V_0 = \rho_0^{-1}$ is the specific volume of the medium ahead of the front; V and p are connected by the equation of the compressibility of the medium ($V = V(p)$).

We go over to dimensionless quantities and to dimensionless Lagrange variables

$$c^* = \frac{c}{c_n}, \quad p^* = \frac{p}{p_n}, \quad \rho^* = \frac{\rho}{\rho_n}, \quad u^* = \frac{u}{c_n}, \quad D^* = \frac{D}{c_n}, \quad t^* = \frac{tc_n}{l_0}$$

for the detonation products

$$x = \frac{h}{l_0\rho_n}$$

for the medium

$$x = 1 - \frac{h - l_0\rho_n}{l_0\rho_0}$$

In the new variables, in the rarefaction wave

$$c^*t^* = (1 - x)^{1/2} \qquad (2.8)$$

$$p^*(t^*)^{3/2} = (1 - x)^{3/2} \qquad (2.9)$$

$$u^* = 1 - \sqrt[3]{p^*} \qquad (2.10)$$

The relationships at the front of the shock wave are

$$u^* = D^*\left(1 - \frac{V^*}{V_0^*}\right), \quad p^* - p_0^* = \frac{3}{V_0^*}D^*u^*, \quad u = \frac{p^*}{3}(V_0^* - V^*) \qquad (2.11)$$

The compression wave in a three-component medium, in dimensionless form, are

$$\frac{V^*}{V_0^*} = \alpha_1\left(\frac{p^*}{p_0^*}\right)^{-\gamma_1^{-1}} + \alpha_2\left[\frac{\gamma_2 V_2^*(p^* - p_0^*)}{3c_2^{*2}} + 1\right]^{-\gamma_2^{-1}} + \alpha_3\left[\frac{\gamma_3 V_3^*(p^* - p_0^*)}{3c_3^{*2}} + 1\right]^{-\gamma_3^{-1}} \qquad (2.12)$$

$$\frac{1}{V_0^*} = \frac{\alpha_1}{V_1^*} + \frac{\alpha_2}{V_2^*} + \frac{\alpha_3}{V_3^*}, \quad c_2^* = \frac{c_2}{c_n}, \quad V_2^* = \frac{V_2}{V_n}, \quad c_3^* = \frac{c_3}{c_n}, \quad V_3^* = \frac{V_3}{V_n}$$

In what follows, we omit the asterisks on the dimensionless quantities.

The initial parameters of a shock wave in a medium, p_T, u_T, and V_T are determined by solution of the system of Eqs. (2.10)-(2.12), expressing the equality of the velocity and the pressure at the contact discontinuity and the law of compression of the medium. These values, calculated for three media, are given in Table 3.

It was assumed in the calculations that

$$\gamma_1 = 1.4, \ \gamma_2 = 7, \ \gamma_3 = 7, \ \rho_3 = 2.65\ \mathrm{g/cm^3}$$
$$\rho_2 = 1\,\mathrm{g/cm^3}, \ \rho_1 = 12 \cdot 10^{-4}\,\mathrm{g/cm^3} \ c_3 = 4500\,\mathrm{m/sec}$$
$$c_2 = 1500\,\mathrm{m/sec} \ \ \rho_n = 1.6\,\mathrm{g/cm^3} \ D_n = 6900\,\mathrm{m/sec}$$
$$c_n = 4200\,\mathrm{m/sec}, \ p_n = 94000\,\mathrm{kg/cm^2}$$

The first medium (Table 3) corresponds to a water-saturated soil with a porosity of 0.4, not containing entrapped air, the second to a water-saturated soil with a porosity of 0.35, containing 2% air, and the third, to water.

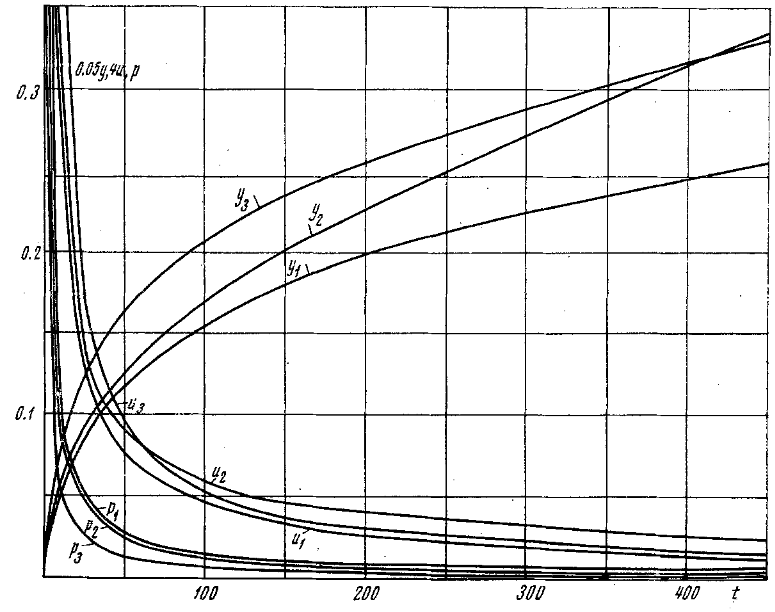

Fig. 4

In the first two media, the initial parameters are practically identical; in water the pressure is less, while the velocity of the particles is greater. This is explained by the greater compressibility of water in comparison with the two first media.

3. Propagation of the Wave

A schematic diagram of different solutions, corresponding to the detonation products and to the medium in the plane xt, is shown in Fig. 2. An analytical solution can be obtained up to the moment of the departure of the reflected wave R_3 to the front of the shock wave. It is advisable, however, to go over to a numerical solution at the moment when the front R_1 has reached the initial cross section, i.e., at $t = 1$. In this case, from (2.8) and (2.9), we find the coordinates of the weak discontinuity R_2 and of the shock front S

$$x_R^{(2)} = 1 - p_T^{-2/3}, \quad x_S = 1 + u_T V_0 (1 - V_T)^{-1} \tag{3.1}$$

At the moment of time in the interval $0 \leq x \leq x_R^{(2)}$

$$p = (1 - x)^{3/2}, \; u = 1 - (1 - x)^{1/2} \tag{3.2}$$

In the interval $x_R^{(2)} \leq x \leq x_S$

$$p = p_T, \quad u = u_T, \quad D = D_T = u_T V_0 (V_0 - V_T)^{-1} \tag{3.3}$$

Further calculation is carried out in an electronic computer using the method of characteristics with fixed time spacing [9]. This method permits determining a solution at points previously given in time and space. In the variables h, t, the characteristic relationships have the form

$$du = \pm \sqrt{-\frac{dV}{dp}}\, dp \quad \text{at} \quad dh = \pm \sqrt{-\left(\frac{dV}{dp}\right)^{-1}}\, dt \tag{3.4}$$

From this, in the dimensionless variables x, t, in the medium

$$du = \pm \sqrt{-\frac{1}{3}\frac{dV}{dp}}\, dp \quad \text{at} \quad dx = \pm V_0 \sqrt{-\frac{1}{3}\left(\frac{dV}{dp}\right)^{-1}}\, dt$$

In the variables x, t, in the detonation products

$$du = \pm {}^1/_3\, p^{-2/3}\, dp \quad \text{at} \quad dx = \pm p^{2/3} dt$$

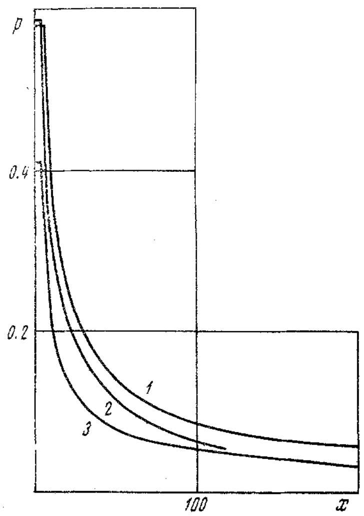

Fig. 5

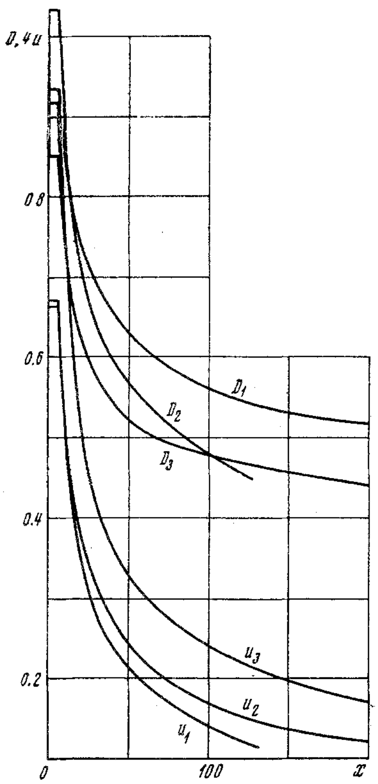

Fig. 6

In the problem under consideration, there are five types of points, at each of which the parameters are calculated in accordance with their own algorithms:

1) in the medium ahead of the shock wave S;

2) in the medium between S and T;

3) at the contact discontinuity T;

4) in the detonation products between T and the initial cross section;

5) at the initial cross section.

To start the calculation, n points are selected in the detonation products, and m points in the medium. At the points selected with t = 1, there enter the parameters determined from (3.2) and (3.3).

Let us consider the sequence of the calculation of points at the front of the shock wave. Points A and B lie on one time layer (Fig. 3). The values of the parameters at these points are unknown. Points B and C lie at the front of the shock wave. The spacing with respect to the spatial coordinate is assumed to be constant

$$\Delta x = \frac{D_T}{2(m-1)}, \quad x_C = x_B + \Delta x$$

The spacing with respect to time varies from layer to layer

$$\Delta t = \frac{2\Delta x}{D_B + D_C}, \quad t_C = t_B + \Delta t$$

In the first time layer $D_B = D_T$. To start the calculation, the values of D, p, and u from point B are carried over to point C. Then, a characteristic curve is dropped from point C onto the preceding time layer. Its intersection with the line AB is designated as L. The coordinates of this point are

$$x_L = x_C - \left[V_0 \sqrt{-\frac{1}{3}\left(\frac{dV}{dp}\right)^{-1}}\right]_{CL} \Delta t$$

$$\frac{dV}{dp} = -V_0\left\{\frac{\alpha_1}{\gamma_1 p_0}\left(\frac{p}{p_0}\right)^{\varkappa_1} + \frac{\alpha_2 V_2}{3c_2^2}\left[\frac{\gamma_2 p V_2}{3c_2^2} + 1\right]^{\varkappa_2} + \frac{\alpha_3 V_3}{3c_3^2}\left[\frac{\gamma_3 p V_3}{3c_3^2} + 1\right]^{\varkappa_3}\right\}$$

$$-(1+\gamma_i)/\gamma_i = \varkappa_i \quad (i = 1, 2, 3)$$

Here, B and C mean that the parameters relate to these points; the subscript CL means that the quantities in brackets are taken as the mean values between C and L. The coordinate x_L found is used to determine the values of p_L and u_L with interpolation with respect to the parameters at the points A and B

$$p_L = p_A + \frac{p_L - p_A}{x_B - x_A}(x_L - x_A), \quad u_L = u_A + \frac{u_B - u_A}{x_B - x_A}(x_L - x_A)$$

Then, the values of p_L and u_L found are used to determine the refined values of u, p, D, and V at the point C, using the relationships at the front of the shock wave and the condition satisfying the characteristic curve

$$p_C - p_0 = \frac{3D_C u_C}{V_0}, \quad D_C = \frac{u_C V_0}{V_0 - V_C}, \quad u_C = u_L + \left[\sqrt{-\frac{1}{3}\frac{dV}{dp}}\right]_{LC}(p_C - p_L)$$

$$\frac{V_C}{V_0} = \alpha_1\left(\frac{p_C}{p_0}\right)^{-\gamma_1^{-1}} + \alpha_2\left[\frac{\gamma_2 p V_2}{3c_2^2} + 1\right]^{-\gamma_2^{-1}} + \alpha_3\left[\frac{\gamma_3 p V_3}{3c_3^2} + 1\right]^{-\gamma_3^{-1}}$$

For purposes of refinement, three to four recalculations are made.

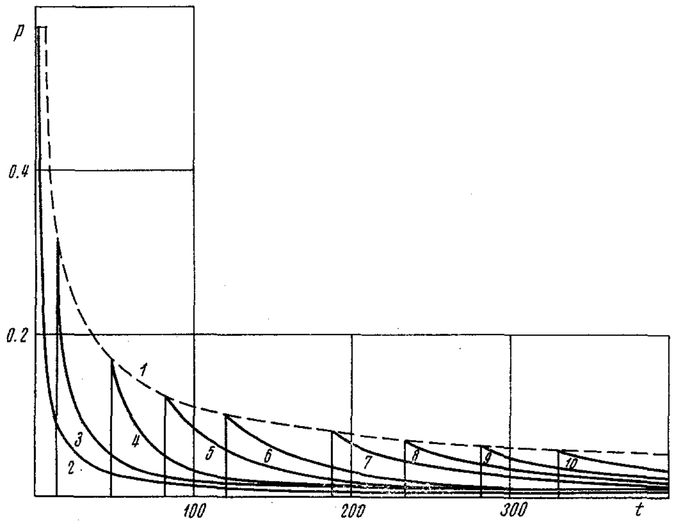

Fig. 7

The calculation is made in an analogous manner for the remaining four types of points, starting from the characteristic relationships in the medium and in the detonation product, as well as from the condition of the equality of u and p at the contact discontinuity.

The calculation was carried out in a BÉSM-4 electronic computer for the three media whose characteristics are given in Table 3. A preliminary calculation was carried out with a number of points in the medium and in the detonation products equal, respectively, to n, m, and 2n and 2m. The difference in the results appeared in the fourth digit.

4. Results of Calculation

Let us consider the parameters at the contact discontinuity T, i.e., at the boundary of the gas cavity. Figure 4 gives curves of the dimensionless quantities, i.e., the pressure p, the velocity of the shock wave, u, and its displacement y, as functions of time. The numbering of the curves corresponds to the numbering of the media in Table 3. In all the media, there was first observed a rapid drop of the pressure and the velocity. At $t > 80$-100, the decrease in these parameters slows down considerably. In the second medium, the pressure falls more rapidly with time, and the velocity more slowly than in the first. The displacement of the boundary in the second medium is greater than in the first. The presence of entrapped air in water-saturated soil leads to an increase in the dimensions of the gas cavity. In a soil with $\alpha_1 = 0$, the cavity is smaller than in water. In a soil containing air, the cavity may be larger than in water.

For equalization of the scale, on Fig. 4 the dimensionless quantities p, 4u, and 0.05 y are plotted along the axis of ordinates. The scale of the dimensional quantities is determined from the condition: $p = 0.1$ corresponds to a pressure of $9.4 \cdot 10^3$ kg/cm^2, a velocity of 105 m/sec, and a displacement of $2l_0$.

Let us consider the parameters at the front of the wave.

Figure 5 gives curves 1-3 for the dependence of the pressure at the front on the distance, in the three media under consideration, and Fig. 6 gives the velocities of the front and the velocities of the particles at the front in the same media. The presence of even a small amount of air ($\alpha_1 = 0.02$) in a water-saturated soil leads to an appreciable lowering of the pressure, the velocity of the particles, and the velocity of the front. With increasing distance from the point of explosion, in a soil with $\alpha_1 = 0.02$, p, u, and D decrease more rapidly than with $\alpha_1 = 0$. This result has been previously obtained experimentally [2]. In water, the pressure and the velocity of the front are less, and the velocity of the particles is greater, than in soil with $\alpha_1 = 0$. With increasing distance from the point of explosion, p and D in a soil with $\alpha_1 = 0.02$ decrease more rapidly and become less than in water.

On Fig. 7, curve 1 corresponds to the change with time of the pressure at the front of the shock wave in the first medium, while curves 2-10 correspond to the change in the pressure at fixed points in space (at particles) with coordinates x equal to 1, 12, 36, 60, 84, 120, 144, 168, and 192. With increasing distance from the point of explosion, the rate of drop of the pressure behind the front decreases, and the time of action of the wave increases. The degree of increase may be characterized by the dimensionless quantity

θ, equal to the time during which the pressure at the point in space under consideration decreases from its maximal value p_m to 0.05 p_m. The corresponding dimensional time $\theta t_0 = \theta l_0/c_n$. The values of θ are given below; θ_1 relates to the first, and θ_3 to the third medium.

x	1	11	12	22	33	36	44	55	60
θ_1	35	—	105			230			295
θ_2	31	80	—	130	185	—	220	260	—

With $\alpha_1 = 0.02$, the value of θ is greater by several percent than with $\alpha_1 = 0$. In all cases, with increasing distance from the point of explosion, the rate of increase in θ declines.

Thus, the wave parameters were obtained in three media near the point of explosion. The calculations correspond to the experimental data [2], showing that, at the front in a water-saturated soil with $\alpha_1 = 0$, p, u, and D have greater values than in water. Even a small ($\alpha_1 = 0.02$) amount of air in a water-saturated soil leads to an appreciable decrease in the values of p, u, D. With increasing distance from the point of explosion, the rate of decrease in these quantities in a soil containing air increases in comparison with a soil where $\alpha_1 = 0$. The time of action of the wave increases with increasing distance.

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